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COMPUTATION OF THE CUBE ROOT OF 2.

BY ARTEMAS MARTIN, M. A., EDITOR OF THE MATH'L VISITOR, ERIE, PA.

HAVING recently computed the cube root of 2 to 52 places of decimals, by the method of approximation found in Simpson's Algebra, I submit it, with the work, for publication.

Let $R =$ the true nth root of a number N, and $r =$ a near approximate root, and put $q = nr^n \div (N - r^n)$; then (Simpson's Algebra, p. 169)

$$R = r + \frac{r(2q+n)}{q(2q+2n-1)+\frac{1}{6}(n-1)(2n-1)}, \text{ very nearly,}$$

which he says (p. 165) "quintuples the number of figures at every operation."

Taking $n = 3$ we have for the cube root of N,

$$R = r + \frac{r(2q+3)}{q(2q+5)+\frac{5}{3}}, \text{ very nearly.}$$

To compute the cube root of 2, take $r = 1.25 = \frac{5}{4}$; then

$$\sqrt[3]{2} = 1.25 + \frac{\frac{5}{4}(253)}{125(255)+\frac{5}{3}} = \frac{5}{4} + \frac{759}{76504} = \frac{96389}{76504} = 1.2599210498+,$$

which is true to the last figure.

Now take $r = \dfrac{96389}{76504}$, then $r^3 = \dfrac{895534711311869}{447767355672064}$,

and after some reductions we get

$$\sqrt[3]{2} = \frac{96389}{76504} + \frac{556917410073276535841 7747}{36812917798558512816995939188473 6464},$$

$$= \frac{463813700424535044109807007546772121}{368129177985585128169959391884736464},$$

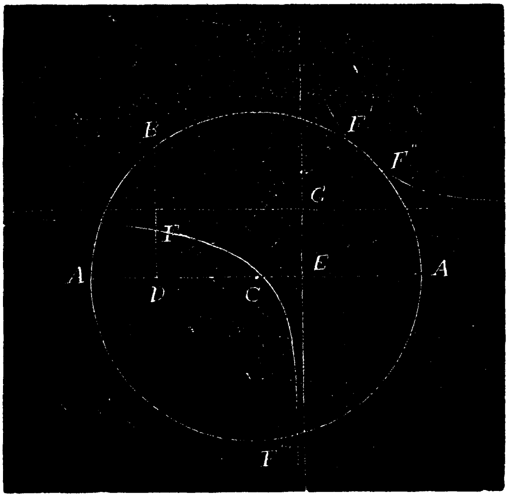
$$= 1.2599210498948731647672106072782283505702514647015079+.$$

ON THE TRISECTION OF AN ANGLE.

BY PROF. J. SCHEFFER, MERCERSBURG COLLEGE, PENN'A.

I SHALL here give some of the different methods which have been devised for the solution of this celebrated problem of the Platonic school.

1. Let $AB = a$ represent an arc, whose radius is r, and let F represent the point in which AB is trisected.



Denoting CD and BD respectively by x' and y', and the coordinates of F by x and y, we have $x'=r\cos a$, $y'=r\sin a$; $x=r\cos\frac{1}{3}a$, $y=r\sin\frac{1}{3}a$.

Since $r^2\sin\frac{2}{3}a = 2r\sin\frac{1}{3}a\times r\cos\frac{1}{3}a$, and also $= r\sin a \times r\cos\frac{1}{3}a - r\cos a\; r\sin\frac{1}{3}a$, we have $2xy = xy'-x'y$, or

$$y=\frac{xy'}{2x+x'}. \qquad (1)$$

Putting $x-\frac{1}{2}x'$ in place of x, and $-y+\frac{1}{2}y'$ in place of y we obtain the simple equation

$$xy=\tfrac{1}{4}x'y', \qquad (2)$$

which represents an equilateral hyperbola referred to its asymptotes.

From equation (2) we can easily construct the curve: Make $CE=\frac{1}{2}CD$, $EG=\frac{1}{2}BD$; through G draw the two axes respectively parallel to AA' and BD, and construct the equilateral hyperbola, cutting the arc AB at F, then will the arc AB be trisected at F.

Combining eqn. (1) with the equation $x^2+y^2=r^2$ of the circle we obtain the biquadratic equation, $x^4+x'x^3-\frac{3}{4}r^2x^2-r^2x'x-\frac{1}{4}r^2x'^2=0$; whose four roots are

$$\begin{aligned} x_1 &= -x' = -r\cos a,\\ x_2 &= r\cos\tfrac{1}{3}a,\\ x_3 &= -r\cos(60^\circ+\tfrac{1}{3}a),\\ x_4 &= -r\cos(60^\circ-\tfrac{1}{3}a), \end{aligned}$$

which shows that there are four points of intersection; x_1 corresponds to the point F'; x_2, to the point F; x_3 to the point F''', consequently $A'F''' = \frac{1}{3}A'F'''AB$; and x_4 corresponds to F'', consequently $AF''=\frac{1}{3}A'F''B$. For $a=45^\circ$, F' and F'' coincide. It follows from this that the hyperbola always trisects both the acute and its supplementary obtuse angle. If therefore an obtuse angle is to be trisected, it is only necessary to trisect its adjacent acute angle.

Pappus was the first who devised and employed the hyperbola for the solution of this problem.

2. Let ABC be the angle to be trisected. [The reader will readily construct this, and the subsequent figures in this Art., from their description.]

Let fall the perpendicular AD; make $CD=2AB$, and describe, with B as pole and AD as base, the upper Conchoid, CE; draw AE parallel to BC and join BE, intersecting AD in F, then is $\angle CBE=\frac{1}{3}\angle ABC$.

For, bisecting FE in G and drawing AG, $CD = FE = 2AB$ according to the nature of the conchoid. Since $FG = GE = AG = AB$, we easily find $\angle CBE = \frac{1}{3} \angle ABC$. By the lower conchoid $C'E'$, the obtuse adjacent angle is trisected. For, bisecting $E'F'$ ($= C'D = 2AB$) in G' and drawing AG', we have $E'G' = G'A = G'F' = AB$, from which at once follows $\angle E'BC' = \frac{1}{3} \angle ABC'$.

Nicomedes devised the conchoid for the trisection of an angle and the duplication of a cube.

3. Let BEC represent an Archimedean Spiral. Divide the radius BC of the circular arc AC, into three equal parts so that $BD = \frac{1}{3}BC$, then, describing from B, with radius BD, an arc which intersects the spiral at E, the angle $ABE = \frac{1}{3}$ angle ABC. For, according to the definition of the spiral, $AB : BE (= BD) = \angle ABC : \angle ABE$.

4. *Montucla* ascribes the following two solutions to the Platonic school.

1. Let ACB be the angle to be trisected. From C, with any radius, describe a semi-circle, and through B draw BE, interseting the circle in D, so as to make $DE =$ the radius of the circle, then angle at $E = \frac{1}{3}ACB$.

2. Let ABC be the angle to be trisected. Complete the rectangle above BC. Produce the upper side, and through B draw BE meeting the upper side produced in E and intersecting the perpendicular CA in D, so as to make $DE = 2AB$, then angle $DBC = \frac{1}{3}ABC$, as can be easily proved by drawing AG to the middle point of DE.

5. The jesuit *Thomas Ceva* devised an instrument for the trisection of an angle. It consists of four rulers, AE, AF, DB, DC, which form a rhombus, $ABDC$, and are movable around A, B, C, D. (The points B, G and C, H being, respectively, on AE and AF.) If the angle GDH is to be trisected, we take $GD = DH = BD$, fasten the instrument at D, and move the rulers so as to make AE and AF pass through G and H, then angle $EAF = \frac{1}{3}GDH$.

6. By approximation we can trisect the angle $BCA = a$, in the following manner:

Make $AD = \frac{1}{4}a$, $DE = \frac{1}{4}AD = \frac{1}{4^2}a$, $EF = \frac{1}{4}DE = \frac{1}{4^3}a$, &c.; then we obtain for the sum of all these arcs, by summing the infinite geometric series $(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots)a = \frac{1}{3}a$.

NOTE ON THE CATENARY, BY PROF. W. W. JOHNSON.—The following formulæ arise in the consideration of the measurement of a base line by means of a steel tape which is allowed to assume the form of a catenary.

The equation of the curve being